International Journal of Scientific & Engineering Research Volume 12, Issue 4, April-2021
ISSN 2229-5518

Review of Economic Statistical Design X-bar Chart with Six Sigma Idea

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Abstract— After Shewhart's suggested first control chart, various chart styles have been created. over time to meet the needs of various manufacturing environments. For the development of Design of control, various methodologies have been developed. The most famous among them are economic statistical design and economic design, which help to minimize the cost of process management in order to compete in a competitive market. It has drawn a lot of interest from researchers that are trying to reduce the cost as much as possible. In the development of economic models, and process characteristics of different types have been considered. For optimizing the design of control charts from an economic standpoint, various forms of optimization statistical techniques such as six Sigma have been tried. This paper provides a summarizing review of the Design of control, statistical concepts of economic design with six sigma SS. Where the six Sigma its prominence in statistical quality control. The capacity process indexes are presents. Finally, the paper concluded with a suggestion to potential researchers to use the six-sigma method on statistical control and during manufacturing processes to reduce defects and variations and manufacture high-quality, cost-effective products.

Index Terms— Six Sigma, Variance, Statistical Process Control, Process Capability, Economic statistical design

1 Introduction

IN today's competitive markets the significant challenge lenge is to provide high quality products and services at at minimum costs. Considering this, the majority of companies are constantly seeking improvements to quality levels and overall performance of manufactured products. To remain competitive, companies are striving to employ efficient strategies and tools to improve quality, minimize defects and variations and produce cost-effective products. Therefore, it is essential to verify that the process performance meet the pre-specified measures to ensure high quality level of products that meet the expectations and demands of customers and management. Quality refers to a product's suitability for use [1]. Quality Control, according to ISO, is a component of quality management that focuses on meeting requirements [5]. Statistical Process Control (SPC) is a collection of problematic-resolving strategies for improving process efficiency and capability by reducing variance with statistical methods [2]. The control chart is the most commonly used of all the SPC methods. It's a graphical method for determining whether a process is under control or not. Shewhart (1931)[3] was the first to introduce the *X* chart to differentiate normal expected causes of process variability from special or assignable causes, and is still widely used today to measure the variability of a given process. Using an X control chart, calculate the values of three parameters: sample size (*n*), control limits *U-L=K*, and sampling interval length (*h*). Following that, several other forms of control charts were created and made available for use in practice. Where there are many types of Control Chart Design: The effectiveness of any control chart's design determines its ability to detect changes in quality level obtained from a process.The statistical and cost properties are also influenced by the design. Samples from the manufacturing process are usually taken at regular intervals to maintain a control chart. A designer must decide how and when a sample will be taken, as well as the size of the sample. He also has to determine how far away from the center line the control limits should be drawn[4]. The sample size refers to the number of objects in each sample (*n*). The sampling interval is the amount of time between two consecutive samples (h). In most control charts, the two control limits are symmetrically positioned from the center line. The width of control limits is the distance between each control limit and the center line, represented as a multiple of the sample statistic's standard deviation (*k*). In certain cases, including the design of an *X* chart, Control chart design refers to the collection of these three parameters, *n, h,* and *k* [2]. Depending on the form of control chart, one or more additional parameters are considered in addition to the above three. In addition to *n, h*, and *k*, a smoothing parameter (*V*) is considered in the economic architecture of an exponentially weighted moving average (EWMA) chart. The four types of control chart design which are detailed below [5].

i. Control charts, In this design, the width of the control limits (k) is set to three, and the sample size (n) is four or five. There is no fixed rule for selecting a sampling interval (h), and the performance rate is a big factor.

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ii. Statistical design: The control charts in this design are rendered on a statistical basis. The statistical errors Type-I error (α) and Type-II error (β) are kept to a bare minimum. The control chart's consumer is in charge of determining these values. The user of the control chart is responsible for specifying these values. Following the correction of regarding these two errors, the next step is to evaluate the sample size and control chart design variables. (n) and the width of the control limits (k). The sampling interval h cannot be calculated using a formula. The control chart's ability to detect a process change quickly is highlighted in this design.

iii. Economic design: First two techniques of design do not take cost into account. So, the cost of inspection and analysis, the cost of untrue alarm, the cost of eliminating the assignable reason, as well as the cost of manufacturing non-conforming product is when process is functioning out-of-control are all heavily influenced by the control chart design variables. To thrive in this competitive environment, all companies must ensure that their overall cost of production is as low as possible. As a result, the design control chart from an analytical standpoint is important [6]. The chart parameters are chosen in this design technique so that the overall cost of control process is held to a minimum.

iv. Economic statistical design: [7] criticized use of economically constructed controller charts, arguing that this form of design lacks the statistical reliability of the control charts. Criticized which is described as keeping Type I and Type II errors to a min-imum. The economic-statistical construction combines both economic and statistical methodologies for the purpose of creating a control chart. Its aim is to lower the total cost of process control while also meeting statistical requirements.

As a result, an analysis sensitivity is obligatory to explore the impact of process and cost parameters on the performance of control chart design. Due to fierce competition in both domestic and foreign markets, today's companies face numerous challenges. For the company's survival and development, the competitive market and consumer knowledge necessitate the production of high-quality products and services. As a result, every manufacture is concerned about the superiority of their products[5].

Simultaneously every company must strive to maximize its profit boundary, which necessitates keeping production costs to the absolute min-imum. In this case, companies have introduced Six Sigma procedure during manufacturing process to reduce defects and variations and ultimately produce high quality products. This paper brings about a concise review of Statistical concepts (concept quality, statistical design, economic, six sigma concepts, and statistical measurements, related in the X-bar chart.) The paper also reviews the related papers and suggested that Six Sigma is significant to be used during manufacturing process to improve quality products. Moreover, the benefit of utilizing Six Sigma tools on statistical control have been addressed in this review paper ranging from the simple variation reduction and defects to the improvement of marketing share and competitive advantage[9].

2 The Concept of Quality and Its Dimensions

Quality Dimensions the quality of a produce can be expressed in a variety of ways. [10] Defined product quality in terms of the eight dimensions mentioned below. When all of these dimensions of quality are correctly balanced while designing and producing a product, it is said to be of good quality. Performance: A high-quality product should meet or exceed the customer's standards. If the product performs poorly, the consumer will be disappointed, and the business will experience a drop in revenue, unfavorable feedback, and a loss of brand goodwill in the long run [2].

Reliability: During its lifetime, a produce must provide reliable and reliable results. When a product fails often, its reliability is considered to be low. Several companies have built their brands and gained consumer confidence by ensuring that their merchandises are extremely reliable.

Durability is a metric that measures how long a product lasts before needing to be replaced. Every product should have a long lifespan.

Serviceability refers to a product's ability to be easily repaired in the event of a malfunction.

Aesthetics: This refers to the product's visual appeal, which is determined by factors such as color, design, form, packaging, sound, and feel, among others.

Features: This consistency dimension refers to the extra features that have been applied to the product's basic operating characteristics.Perceived efficiency: This quality dimension is related to the company's or products previous credibility. In the view of the consumer, it relates to the understanding of the product's consistency.

Conformance to requirements refers to the extent to which a product complies with its stated specifications. The product must adhere to existing national or international standards.

2.1 Engineering of High Quality

Quality engineering is a collection of engineering, organizational, and administrative practices used by a company to ensure that a product's quality characteristics are at the required levels and that across variability these levels is kept to a least [2].

2.2 Management of Quality:

The overall activities and tasks needed to sustain quality at a desired standard of quality are recognized as an organization's quality policy [53]. The overall administration role that determines and implements the quality policy is known as quality management [53].

2.3. Quality Assurance

Each customer has an impression about the product's quality and cost, even if he doesn't know how to describe them correctly. It is the manufacturer's duty to research the customers' requirements in depth, analyze their suggestions, and make every effort to create a product that meets the customers' requirements satisfactorily. Quality assurance refers to any of the expected or systematic steps that must be taken to ensure that a product will meet a set of requirements [53].

2.4 Aspects of Quality

Generally, there are 3 quality aspects correlated with the concept of quality assurance, namely I design quality, (ii) conformance quality (iii) performance quality [53]. The tightness of requirements for producing a product determines the product's design efficiency. Products are produced in a variety of quality ranges. Product type, cost, profit policy of the business, demand for the products, material obtainability, and product-safety all influence quality levels. Higher design quality usually entails a higher price, but it also sometimes entails a higher value.

2.5 Quality Control

The process of measuring real quality output, comparing it to expectations, and taking corrective measures if there is a deviation is known as quality control (QC)[2]. Inspection, data analysis, disorder analysis, remedial behavior, salvage (i.e., scrap or rework) decision methods, vendor relationships, and the institute of quality standards are all major QC roles.

2.6 Quality Characteristic

Quality characteristics are the characteristics that are used to assess the quality of a product. These are often referred to as critical-to-quality (CTQ) products[2]. In general, quality characteristics can be divided into two groups, as shown below:

I. Variable data: Variable data refers to quality attributes that can be quantified and measured. To put it another way, these are calculated as numbers on a continuous scale of measure, such as length, weight, volume, and so on. Continuous data is another name for them[2].

II. Attribute data: Attribute data is a concept used to describe intangible attributes that cannot be calculated on a continuous scale. They are classified as either conforming to specifications or not conforming to specifications. Discrete data is another name for them [2].

Data obtained through actual calculation is vector (continuous), while data obtained through counting is attribute (discrete). The current research focuses on control charts with variable data.

2.7 Variation's Causes

Even if great care is taken to render two items similar, the law of variance states that no two items can be exactly identical. Variation is a natural occurrence. This is true of production methods as well. A certain quantity of inherent normal variability occurs in any manufacturing phase, regardless of how well it is engineered or managed. Chance (or common) cause of variation refers to any form of variation that occurs as a result of a combination of various inevitable causes of small magnitudes [2]. Statistical control refers to a mechanism that operates with only chance causes of variance. In this case, the mechanism is said to be in order. These factors, according to Shewhart (1931), occur on a continuous basis, are difficult to detect or eradicate economically, and do not result in any improvement in quality standards. Another form of variability, on the other hand, has characteristics such as a larger magnitude of change in quality level, sporadic occurrence, simple identification, and cost-effective removal. Assignable (or special) causes of variance are the names given to these sources[2]. Three Ms may be one of the main causes of this variability (i.e., man, machine and materials). Out-of-control processes are those that operate in the attendance of both assignable triggers.

3. STATISTICAL PROCESS CONTROL

(SPC) is a collection of problematic-solving methods for attaining process reliability and enhancing capability by using statistical approaches to reduce variability[2]. The control chart is the most commonly used of all the SPC methods.

3.1 CONTROL CHARTS

A control chart is a graphical impersonation of data gathered from samples-taken from a procedure at various points in time. Shewhart (1931) was the first to use it. Shewhart control charts are commonly used to establish control statistical over a production process in a variety of fields.

The control chart is a two-dimensional graph with a horizontal axis that signifies sample collection time or order and a vertical axis that represents sample statistic. A Shewhart control chart normally has 3 horizontal lines, one for the center line (CL) and two for the lower and upper control limits (L and U). If there are no unusual sources of uncertainty, the process characteristic is supposed to fall on the center line. The sample mean will plot beyond the control limits if this is present (i.e., either UCL or LCL). The control chart offers clear indication that the mechanism might have went out of control due to some reason if a point falls beyond one of the two control limits. The control chart must be designed in such a way that it can generate the signal as soon as the process has gone out of control. The delay in causing an out-of-control signal would cause the company to lose money by creating more and more non-conforming products. When there is a delay, the quality of the product can deteriorate further, resulting in a loss of higher magnitude.

The control chart must be designed in such a way that it can generate the signal as soon as the process has gone out of control. The delay in causing an out-of-control signal would cause the company to lose money by creating more

and more non-conforming products. It's possible that the delay will worsen in the future.

In any control chart, two types of judgment errors are committed because the decision about the process status is based on the results obtained from a small size sample data. When the mechanism is still in order, but the control chart shows that it has gone out of control, a Type-I or error has occurred. Type-II or malfunction, on the other hand, occurs when the control chart is unable to provide a warning when the mechanism has really gone out of control. The greater the power of detecting process change, the lower the value of error[2].

This chart is still widely used in practice, and there is a lot of study on the design aspect of it in the literature. It's only meant to be used as a diagnostic tool. It only means that a mechanism has become uncontrollable. It is unable to self-correct an out-of-control mechanism and return it to an in-control state without the assistance of quality staff. The primary goal of a control chart is to quickly locate the occurrence of an assignable cause in a production process that has gone out of control, so that the process of hunting for and eliminating assignable causes can begin[5]. If there is a delay in identifying the assignable cause, there will be a delay in taking corrective steps, resulting in the mass production of non-conforming goods. Furthermore, investigating and correcting a process after receiving a warning from the control chart necessitates engineering knowledge of the manufacturing process. When a process is monitored using a control chart, too much monitoring results in additional costs, and too little monitoring results in quality issues. As a result, maintaining process control at the lowest possible cost is important. Control charts with a low cost of design aid in achieving this goal. Until development may begin, two conditions must be met in order to minimize the proportion of non-conforming products [11].

i. The mean of the quality characteristic's distribution should be as similar to, if not exactly equal to, the target value.
ii. The variance of the quality characteristic distribution should be as low as possible.

Allow the mean and standard deviation of a quality to be determined. The characteristics of an in-control mechanism are denoted as. The process mean and process standard deviation should always be equal to the in-control mean (μ_0) and in-control standard deviation (σ_0), espectively. To accomplish this, the process is tracked by taking samples at regular intervals and making inferences. This is statistically similar to hypothesis checking. The two hypotheses that must be tested are as follows:

$$1)\ H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$
$$2)\ H_0 : \sigma = \sigma_0$$
$$H_1 : \mu \neq \sigma_0$$

When H_0 is acknowledged in process control, the process is believed to be in control and permitted to run. If H_0 is dismissed, the mechanism is believed to be out of reach, and it is investigated to determine the assignable triggers. It is necessary to eliminate the assignable triggers after they have been detected. Since this protocol must be followed before the manufacturing process is completed, it is more convenient to depict it on a control chart. The control chart is a two-dimensional graph with a vertical axis representing sample statistic and a horizontal axis representing sample collection time on a two-dimensional graph.

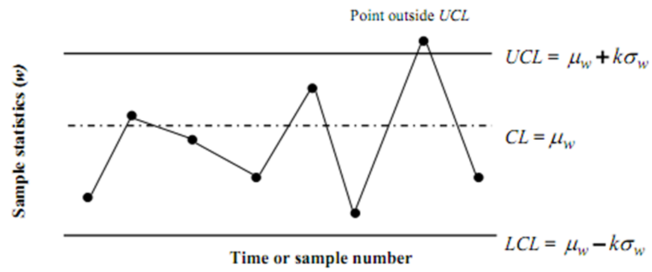

Fig. 1: Shewhart control chart

These three lines are determined as follows if W a sample statistic In a general model for a control map, is determined from all measured values of a quality charac-teristic of interest for all the products in a sample.

$$CL = \mu_W$$
$$UCL = \mu_W + K\sigma_0$$
$$LCL = \mu_W - K\sigma_0$$

When the method is under control, k is the standard deviation units used to express the width of the control-limits, μ_W and σ_W is the mean and standard deviation of the sample statistic. w When a point on a control chart goes either above or It means the mech-anism is no longer controversial since it is below the upper or lower control limits.

3.1.1 Control Chart Types

The type of quality characteristic being considered divides control charts into two groups: Control charts can be divided into two categories: (i) variable control charts and (ii) attribute control charts. Variable monitoring charts include the range ($\bar{R}$) standard deviation (σ), and mean ($\bar{X}$) sample population, S.D sample is (S_2), and population S.D is (σ^2). Fraction non-conforming (p), is (np), non-conformity (c), and non-conformity per unit are all examples of attribute control charts (u).

Control charts are categorized as i univariate charts or ii) multivariate charts based on the number of consistency characteristics. Multivariate charts deal with more than one consistency characteristic, while univariate charts deal with only one. Hotelling T^2 and MEWMA (multivariate exponentially weighted moving average) charts are examples of multivariate charts. The current study is concerned with univariate $\bar{X}$ and $\bar{R}$ charts. Depending on the sampling interval, a control chart may be categorized as a fixed or variable sampling interval chart. In a fixed sampling interval control chart, the time interval between two consecutive samples is set[5]. The length of the time interval between two consecutive samples in variable sampling charts varies depending on the frequency of the impact of assignable variables. In this analysis, set sam-pling interval control charts were used.

I. Error Types:

The hypothesis test theorem is used to build a control chart. The effects of a small sample of data are used to make a decision about the procedure. In any control chart, there are two types of judgment errors that can be made. When a mechanism is still under control but the control chart shows that it has become uncontrollable, α error or Type-I has occurred [2]. This is often referred to as a false-alarm. The false alarm is caused by the control-chart when there is no assignable cause. As a result, a false alarm leads to an unnecessarily extensive quest for assignable triggers. Type-II or β error However, this can happen if the control chart is helpless to tool up an out-of-control caution when the process has really wasted out of control [2]. The incapability to identify any process alteration in real time results in a lack of quality control. In addition, if the β error rate is high, a greater number of non-conforming products will be generated. When a mechanism has gone out of balance, the probability of detecting the assignable trigger (P) is referred to as power (P), and it is expressed mathematically as ($p = (1-\beta)$ 1). As a result, the lower the error value, the faster the rate of process change detection. As a result, a control chart must have as few α and β errors as conceivable [2]. Figure 2 depicts the 2 errors for a Shewhart control chart.

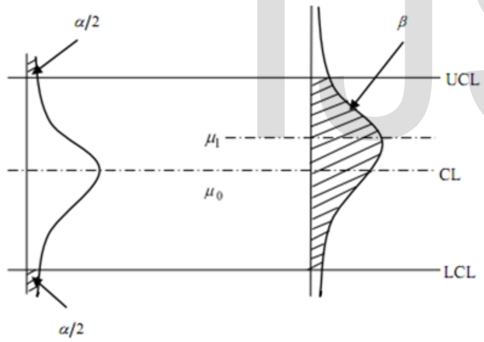

Fig. (2) Shewhart control graph, shown two types of errors.

The average run length is the estimated number of sam-ples taken before a sample point on the control chart gen-erates an out-of-control signal, that is, a sample point reaches one of the 2 control limits (ARL) [5]. When the procedure is denoted as ARL_0, while the mechanism is under control, ARL is used, and when it is out of control, ARL is used. These two ARL_0 can be expressed in a variety of ways.

Cita $ARL_0 = 1/\alpha$ and $ARL_1 = 1/(1-\beta)$

In addition, the average or mean time signal (ATS) is the assessed time it would take to get the first indication that the mechanism has really gotten out of control. $ATS = h \times ARL$ ARL is studied by multiplying the interval sampling (h).ATS is based on sample size n and control limit width k, while average run length is a function of all 3 variables n, h, and k. There are 2 average times to signal, ATS_0 and ATS_1, much like ARL_0 and ARL1. When a process is in control, ATS_0 is the average time it takes for it to signal that it has lost power. ATS_1 is the average time it takes for a system to warn that it has gone out of control when it is really out of control. Is the typical period As a result, ATS_0 should be elevated at all times, while ATS_1 should be low at all times. The following are the results of measuring each of these *ATSs*::

$$ATS = h \times ARL_0 = \frac{h}{\alpha} \text{ and}$$

$$ATS_1 = h \times ARL_1 = \frac{h}{1-\beta} = \frac{1}{p}$$

3.1.2 X and R Charts:

Because of its simplicity, the $\bar{X}$ chart is the most commonly used control chart in industry for tracking and managing processes. However, it can only detect process changes caused by a shift in the process mean. A method, on the other hand, can become uncontrollable due to a change in the process mean and/or variability. As a result, it's common to suggest using both $\bar{X}$ and R charts for statistical process management. Factor data is used in both of these charts. When more than one control chart is used, economic design and economic statistical-design are referred to as joint eco-nomic design and joint economic economic-statistical de-sign, respectively. Almost all manufacturing processes have a normal distribution for the quality characteristic $\bar{X}$. For example, $\bar{X}$ may be the bolt's external diameter or the nut's internal diameter. $\bar{X}$ Is the trait that has the greatest impact on the product's efficiency. In a sample of size n, values of X are computed for all of the elements, and Sample mean $\bar{X}$ is the average of these values, which is calculated as

$$\bar{X} = \frac{X_1 + X_2 + ...X_n}{n}$$

The samples are taken at predetermined intervals. One sample mean $\bar{X}$ is measured for each sample. Since $\bar{X}$ is plotted, this control chart plot is called $\bar{X}$ chart. If $\mu \& \sigma$ for an in-control process are the process standard deviation and mean of a quality characteristic X , and both are calculate as follows:

$$CL_{\bar{X}} = \mu_0$$

$$UCL_{\bar{X}} = \mu_0 + K\frac{\sigma_0}{\sqrt{n}},$$

$$LCL_{\bar{X}} = \mu_0 - K\frac{\sigma_0}{\sqrt{n}},$$

When these two performance parameters μ_0, σ_0 are unknown, When the process is under control at the start, A minimum of 20 to 25 preliminary samples are collected [5]. When (μ_0) preliminary samples of size (n) are taken, the grand mean of sample means, which is expressed as following

$$\mu_0 \approx \bar{\bar{X}} = \frac{1}{n}\sum_{i=1}^{n}\bar{X}_i$$

The value of $\bar{\bar{X}}$ is used as the middle line on the $\bar{X}$ chart. Similarly, to approximate the value of (σ), the standard deviation. Due to the small sample size, there isn't much of an efficiency loss when measuring (σ) from the sample ranges. In addition, estimating the sample range is much easier. If the lowest and highest values in any sample are X_1 and X_2, then the range R is determined as $R = X2 - X1$. The average range $\bar{R}$ is determined using n number of initial sample data as follows:

$$\bar{R} = \frac{1}{n}\sum_{i=1}^{n} R_i$$

$W = R / \sigma_0$, also known as the relative range, is a random variable since R is a random variable. W has a mean of d_2 and a standard deviation of d_3 in its distribution. As a result, $\mu_R = d_2\sigma_0$ and $\mu_R = d_3\sigma_0$ d3S Rd. For a given value of sample size n, the values of d_2 & d_3 are constants. Any statistical quality management text book, such as Montgomery's, has a table of d_2 & d_3 values for different values of the sample size n Montgomery (2013). μ_R Can be calculated as $\bar{R}$ because the process parameters are unknown. As a thus, $d_2\sigma_0 = \mu_R \approx \bar{R}$ and σ_0 can be calculated as:

$$\sigma_0 = \frac{\bar{R}}{d_2}$$

As a result, if the values of (μ_0) and (σ_0) are substituted for all three horizontal lines in the $\bar{X}$ chart for an unknown phase that the following can be written as the following:

$$CL_{\bar{X}} = \bar{\bar{X}}$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + \frac{k}{d_2\sqrt{n}}\bar{R}$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - K\frac{\sigma_0}{\sqrt{n}},$$

Three sigma thresholds are used in the majority of situations in use. As a result, by setting $K = 3$ a new constant A_2 both control limits can be simplified can be defined as the following:

$$UCL_{\bar{X}} = \bar{X} + A_2\bar{R}$$

$$LCL_{\bar{X}} = \bar{X} - A_2\bar{R}$$

The values of constant A_2 are tabulated for various sample sizes n in any statistical quality control text book, such as [2] Montgomery's (2013) R Charts are the same as X charts. The only difference is that the sample range R is plotted along the y-axis instead of the X-axis. Three horizontal lines are also present. There are three horizontal lines here as well. Substituting the value of $\sigma_R = d_3\sigma_0 = d_3\frac{\bar{R}}{d_2}$ for the method with unknown parameters, the three-lines are expressed as:

$$CL_R = \bar{R}$$

$$UCL_R = \bar{R} + \frac{kd_3}{d}\bar{R} = (1 + \frac{kd_3}{d_2})\bar{R}$$

$$LCL_R = \bar{R} - \frac{kd_3}{d}\bar{R} = (1 - \frac{kd_3}{d_2})\bar{R}$$

Using two constants for 3 sigma-limits (i.e., k = 3).

$D_3 = (1 - \frac{3d_3}{d_2})$ and $D_4 = (1 + \frac{3d_3}{d_2})$ As shown below, the two control limits can be simplified.

$$UCL_R = \bar{X} + D_3\bar{R}$$

$$LCL_R = \bar{X} - D_4\bar{R}$$

The values of constants D_3 and D_4 can be found in any statistical quality control text book, such as Montgomery's, (2013) for different sample sizes n. R with a negative value is meaningless. If the measured value using the above expression is a negative number, the lower control limit LCL_R is set to zero. After the three lines have been formed, the values of X & R for each sample are determined and plotted on the appropriate charts. When a point on a chart falls above or below the UCL or LCL, it means the process has lost control, and the necessary steps are taken to examine and remove the causes so that the process will return to normal. If the point, on the other hand, is within both control limits, the system is presumed to be under control and can continue as is. As a result, these two control charts aid in the smooth operation of the operation.

3 Control Designs

4.1 Control Chart Design

Parameters including sample size (n), sampling interval (h), and control limit distance (k) influence the performance of a Shewhart control chart. There are various types of control charts, each with its own set of parameters. Control chart design is the process of choosing suitable values for these control chart parameters [2].

The width of the control limit is the only factor that decides the likelihood of making a mistake (k). As the control limit's range is reduced, the α error increases. The error, on the other hand, is determined by the sample size (n) as well as the width of the control limit k. (k). Any change in the values of these two design variables would almost certainly affect the β error value.

The greater the power of detecting a process change, the lower the value of β error. With a control chart, the capacity of identification of an out-of-control phase can be improved in three ways: i raising the sample size (n), (ii) shortening the sampling time (h), and ($iiii$) narrowing the width of control limits (k) [12].

According to the first approach, if the number of items inspected is larger, small size changes would be easier to detect, but inspection costs would be higher. The second approach increases the power of detecting change by shortening the sampling interval, but the samples must be inspected more regularly, increasing the inspection cost. In the third method, narrowing the width of control limits aids in early detection of rocess change, but it also raises the rate of false alarm (i.e., α error), resulting in wasted time in looking for them. However, it reduces the rate of true out-of-control signal generation (i.e., β error) at the same time. In this scenario, the process engineer is prone to dismissing the true warning, believing it to be a false alarmControl chart parameter selection is important because it can affect the design's expense, statistical properties, and, ultimately, the user's confidence. For different forms of control charts, there are a number of design methods [13] Grouped control chart design into four categories. Heuristic design, statistical design, economic design, and economic statistical design

4.2 Heuristic Design

Shewhart control charts are often used in practice because they are simple to understand and incorporate with minimal operative instruction. In the case of an X-bar chart, the Guidelines for heuristic design of X-bar chart such as Burr (1953) $n = 4 \text{ or } 5, h = ?\ \&\ k = 3$ Feigenbaum (1961) $n = 5, h = 1 \text{ and } k = 3$ Juran et al. (1974) $n = 4, h = ? \text{ and } k = 3$ and [14] Ishikawa (1976) $n = 5, h = 8 \text{ and } k = 3$ where a question mark (?) indicates that no specific value has been defined for the sampling interval selection (h). All of these guidelines use 3-sigma control limits (i.e., $k = 3$), but this design lacks sufficient power to detect process shifts, particularly for smaller size shifts.

4.3 Statistical Design

This design by statistical principles, in which neither the nor the errors are allowed to exceed the user-specified limits. Woodall (1985) estimated two control chart design variables based on these two errors: the width of control limits (k) and the sample size (n). Various authors have chosen different values for, α error (ARL_0) and error β (or, ARL_1), but no general guidelines for different output circumstances have been considered. There are no analyses or instructions for determining the significance of the sampling interval, which differs from one individual to the next. Control chart design and error probability collection, on the other hand, are not economically justified. [15] Keats et al. (1995) created a mathematical model for design an X-bar chart with sampling average and max-imum untrue alarm rate as inputs. The ratio of the number of units sampled in a time interval to the quantity generated in the same time interval is known as the sampling rate. In the case of joint statistical design of X bar and S-bar charts for minimization of sample size n by taking ARL constraints, [16] Wu et al. (2002) developed a computer program written in C. language. For the joint statistical design of adaptive X and R charts,[17] De-Magalhaes (2006) used a Markovian approach. Two separate assignable triggers are applied to the operation. One cause affects the process mean, while the other affects the process variance. [18] found that when developing a Hotelling T^2 chart for a specific value of average time to signal and modified mean time to indication, the adaptive variable parameter Hotelling T^2 chart outperforms the fixed sampling rate (AATS). The following are the various charts in which statistical designs were reviewed by previous researchers: [7] X–bar and CUSUM, [15], [16] Wu et al. (2002) X-bar and S-jointly and [18] Chen (2007) Variable parameter T^2 chart

4.4 Economic Design

Lorenzen [19] suggested a single model that could be generalized to most forms of control charts that work or shut down when searching for and laminating assignable triggers.The use of an economic approach to optimize X-bar chart parameters has been extensively discussed in the literature [6], [8], [20]–[23] Centered on an expansion of [24] total cost model, [25] proposed an econ omic design model with variable sample size.However,Woodall, (1986) noted that an economically built control chart produces a large number of false alarms. As a consequence, when designing the control chart, the goal should be to minimize the cost function while adhering to certain statistical constraints. This is known as the eco-nomic-statistical control chart style, and it was first suggested by Saniga, (1989) to combine economic and statistical objectives. However, it is more expensive than a simple economic design. [26] has written a comprehensive analysis of the literature on this style of design. Researchers such as [4], [23], [27]–[30] have used various optimization methods for this design . Quality and cost have become crucial variables that manufacturers must recognize in conventional *X* Production [4]. Companies are currently facing major challenges in staying competitive and delivering high-quality goods while maximizing profit and lowering operating costs. The majority of industries and manufacturing firms strive to enhance their products' quality and efficiency.This rivalry has prompted businesses to use statistical quality management techniques and strategies in order to increase product efficiency [31][32]. Statistical methods have been used to measure and improve product quality since the invention of the ordinary least squares system. Carl Friedrich [33] (1777–1855), for example, introduced the natural curve principle as a norm for measuring variations. A manufacturing process, in general, works in an in-control mode, generating an output product over a long period of time. Assignable triggers, on the other hand, happen at random and cause the process to go out of control, with a significant proportion of the process output not meeting the requirements.

4.5 Economic Statistical Design

Saniga (1989) [13] was first to introduce the economic statistical-design, which was inspired by Woodall's (1986, 1987) critiques of the shortcomings of economic control chart design. The aim of this form of design is to min-imize the cost function while keeping some statistical con-straints on both α & β errors in mind. This incorporates aspects of both economic and architecture of statistical. This form of design is more expensive than economic design due to the imposition of constraints [34]. However, since it improves the statistical efficiency of the control table, the extra cost is justified [13], [35] contrasted the efficiency of the X chart's economic, mathematical, and economic statistical designs. Centered on Lorenzen and Vance's unified method, [2], [36].
Montgomery (1995) suggested an EW-MA chart as an economic design for tracking the process mean under statistical constraints (1986). They compared economic and economic statistical design and discovered that applying statistical constraints to the cost model resulted in a significant cost increase. This cost increase can be justified if the chart's statistical efficiency increases significantly.They also proposed that, for a small cost increase, any number of restrictions on the out-of-control ARL could be applied for better security. [37] Proposed a restricted cost model for adaptive X-bar charts with dual sample size and dual sampling interval for adaptive X-bar charts. Morales (2013) considered general failure distribution in his integrated model for economic statistical design of joint Xbar and Sbar charts with preventive maintenance. [25] Amiri et al. (2014) suggested the Taguchi loss function method for adaptive X-bar map economic statistical design and compared the results to those of a fixed sampling strategy. [39] Used an economic statistical design of an Xbar chart when the quality characteristic has a non-normal symmet-ric distribution. He proposed the studen, standard Laplace, and logistic distributions as non-normal symmetric distributions and compared their results to the hat of normal, Pearson, and Johnson distributions.
A control chart's main goal is to quickly identify assignable cause(s) so that the process can be investigated and corrective action taken before a large number of nonconforming units are generated. Before using a Shewhart control chart, the sample size n, the time interval between samples h, and the control limits k must all be selected. It is assumed that n is dimensionless, k is a multiple of the standard deviation of the statistic plotted on the table, and h is in hours. The selection of the control chart's configuration (*n, h, and k*). [40] According to the Poisson distribution, Duncan (1956) calculated the overall net income of a process being monitored by an X-bar chart when the process is subject to a random shift in the process mean due to the occurrence of a single assignable cause at a rate of (λ) per hour. Although the assignable cause is being investigated, the mechanism is not shut down, and the cost of returning power to the process is not factored in.. The process is assumed to begin in i, n a statistical control condition, with the mean (μ) and standard deviation σ. The control chart's center line is held at the process's mean (μ_0), the uppe and lower control limits of the process are.

$$\mu + k\,\frac{\sigma}{\sqrt{n}} \quad \text{and} \quad \mu - k\,\frac{\sigma}{\sqrt{n}}$$

False alarms can occur at a rate of α when the process is under control, which is the reciprocal of the average run length under control and is expressed as:

$$\alpha = 2\int_{k}^{\infty} \phi(z)dz$$

Where Φ (z) denotes the normal Probability density. According to the exponential distribution, the mechanism may be interrupted at random by an assignable trigger occurring at a rate of L. The following formula can be used to calculate the strength of detecting process shift in every subsequent sample: The power of detecting process change in any subsequent sample is: If the shift in process mean is λ, the power of detecting process shift in any subsequent sample is:

$$p = 1-\beta, = \int_{\infty}^{-k-\delta\sqrt{n}} \Phi(z)dz + \int_{-k-\delta\sqrt{n}}^{\infty} \Phi(z)dz$$

The hunt for the assigna because begins without stopping the process once the control chart defines the shift in the process mean by plotting a point beyond either the upper or lower control limit. A development cycle is divided into four stages:

I. in control
II. The time it takes to produce an out-of-control signal
II. The time it takes takes to produce an out-of-control signal
IV. The quest for and removal of assignable triggers.

4 Six sigma concepts

Six-Sigma is a modern method that has piqued the interest of researchers, especially those who work in multinational corporations. For example, a specialist engineer named Bill Smith founded Six-Sigma for Motorola Company in early 1986. [41] Smith, the founder of Six Sigma, looked at the differences in outcomes that occur in the company's internal processes. He noticed differences in the internal processes of the company's operations and attributed them to human error. He emphasized the possibility of enhancing a system's efficiency by lowering errors [42]–[44]. Furthermore, Six-Sigma principles use statistically well-proven techniques and methods such as (SPC) and design of experiments, as well as statistical thinking, to help minimize defects and variability in processes [42], [45]. For minimizing variance in processes, this approach heavily relies on advanced statistical methods that work in tandem with product and process information [46]. The word "Six Sigma" comes from the statistical field of process capacity indices, and it refers to an efficient system that creates substantial outputs within the process specification while maintaining six-sigma efficiency, and the outcomes produce long-term errors of less than 3.4 per million opportunities (DPMO).

Six-Sigma implicitly seeks to upgrade whole processes, and not inherently to a 3.4 DPMO level, because industrial organizations are continually striving to develop and achieve sigma levels for their manufacturing processes in order to stay competitive by promoting key specific area developments [47]. It reflects the variability in any operation's intensity. Data that deviates from the mean (arithmetic deviation), as well as data with a standard distribution form, is deviated from the mean (arithmetic deviation). Six Sigma is a statistical indicator that displays the standard deviation of a dataset based on six dimensions or standard deviations [48]. To be more precise, it means reducing (errors) variance to six standard deviations in the efficient phase.

It represents the variation around the rate of any operation. Data is deviated from the mean (σ), which means the arithmetic deviation, not to mention the data that takes the normal distribution shape. To be more accurate, it indicates to reduce (errors) variation to six standard deviations to the productive process, i.e. Six Sigma is a statistical measure, which shows the standard deviation of certain dataset based on six dimensions or standard deviations [48]. Meanwhile, the Six-Sigma approach is used to define processes, evaluate them, monitor them, and develop them in order to minimize defects and uncertainty [49], [50].

5.1 Six Sigma with $\bar{X}$ control chart

When the process parameters, μ — mean and σ — standard deviation, are unknown in statistical quality control based on Shewhart 1931, they are calculated as $\hat{\mu}$ and $\hat{\sigma}$ from sample data obtained from the refinery. The conventional control limits $\hat{\mu} \pm 3\,\hat{\sigma}/\sqrt{n}$ of the study, according to Schoonhoven et al. (2009), provide different performances than the control limit $\mu \pm 3\sigma/\sqrt{n}$ employed for sample means. Control limits can be corrected in this case by replacing the set constant 3 with $\hat{\mu} \pm c(n,k,h)\hat{\sigma}/\sqrt{n}$ and $\hat{\mu}\ \mathrm{m}\, c(n,k,h)\hat{\sigma}/\sqrt{n}$ control limits, where $c(n,k,h)$ denotes the dependent factor on the number of out-of-control signals h. We plan to use 4.5 sigma to estimate the upper and lower control limits of the oil refining process in this study because 3 sigma estimation is inadequate. Despite the fact that there are a number of estimation approaches, studies have shown that 4.5 sigma is efficient at reducing variance and thus improving process efficiency [51], [52]. Control limits can be determined using the following mathematical expression, given k as the sample size of n and 4 items with means $\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots \bar{x}_k$

$$\hat{\mu} \pm z\alpha(k)\,\hat{\sigma}_{SS}/\sqrt{n}$$

where $\hat{\mu} = \frac{1}{k}\sum_{i=1}^{k}\bar{x}_i$, $\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n}x_{ij}$,

x_{ij} is the jth observation of ith sample

$$P\left[(-Z_\alpha(k) \le Z \le (+Z_\alpha(k)\right]1-\alpha_k$$

The regular normal variate is denoted by the Z. The current Sigma Quality Level (SQL) at which the process should be managed is represented by k . If k = 6, for example, the DPMO 3.4 is either on the left or right tail. As a consequence, $\alpha_{kc} = (6.8)10^{-6}$ implies $Z_\alpha(k) = 4.5$, in (1), $\hat{\sigma}_{SS}/\sqrt{n}$ is the approximate standard deviation associated with $\hat{\mu}$ from the Six Sigma Consistency perceptional definition (SSQ). The following procedure can be used to obtain $\hat{\sigma}_{ss}$ from the perspective statistical SSQ: assume that X is a measurable characteristic, so the normal process consists of mean $T = \mu$ and variance σ^2. Meanwhile, the measurable characteristic denoted by X includes values in the form $T \pm k\sigma$, where T denotes a target or population mean, k denotes a positive constant, and σ denotes the population standard deviation.

As a result: X~ $N(T,\sigma^2)$ *and* $p\ (T - k\sigma \le X \le T + k\sigma.) = 1 - \alpha_k$,

where α_k is the predetermined probability value in which $\alpha_k = p(X < T - k\sigma) + p(X > T + k\sigma)$. Based on $T \pm k\sigma$ half of the process spread can be calculated as $k\sigma = d$ indicating $\sigma = d/k$, thus, $\hat{\sigma}_{SS} = d/k$ and we have $\frac{\sigma_{SS}}{\sqrt{n}} = \frac{d/k}{\sqrt{n}}$ The (d) is set according to this equation; as the constant (k) increases, the Sigma(σ)decreases, and vice versa. As a consequence, the constant k is the process' SQL in terms of the quality characteristic X. The specification limit $T - k\sigma$ is the LSL, and the specification limit $T + k\sigma$ is the USL. As a result, the required Six Sigma-based control limit for a typical SSQ process is: $k = 6$ and $\hat{\sigma}_{SS} = d / 6$.

$$\hat{\mu} \pm (4.5)\left(\frac{\hat{\sigma}_{SS}}{\sqrt{n}}\right) = \hat{\mu} \pm (4.5)\left(\frac{d/6}{\sqrt{n}}\right)$$

5 CONCLUSION

Various methodologies have been advanced for the development of the Design of control, and this research article has been set up to evaluate and compare the SPC methods that have previously been published in the literature. The most common among them are economic design and economic statistical design, which help to minimize the cost of process management in order to compete in a competitive market. We acknowledged the obvious need for a coherent system in which these Designs of control based on six sigma techniques could be applied to project control after a thorough discussion of these approaches. An analysis of variance is used to examine the impact of model parameters and their interactions on the optimal solution using a six sigma model. The Six Sigma approach contributes to the development of high-quality goods with minimal defects and process variations in industries. Specifically, unlike the traditional control chart, the six sigma with economic statistical design chart supports in determining the status of the method in terms of level sigma and DPMO of process. This, in essence, helps in involving in ongoing quality assurance activities, even if the sample mean is a known quantity, so that the method is an unbiased estimator of the population mean. Will be able to meet the Six Sigma quality target. As a result, a practice of estimating population standard deviation over time, such as sample standard deviation, sample range, and so on, has emerged. While this study does not claim to be comprehensive, it does include useful information about the current state of economic statistical design analysis. One of the most important findings from our research was the increased analytical emphasis on economic statistical architecture, which has shifted to the use of Six Sigma tools in recent years. There is very little space in the literature for resolving the ambiguity on what constitutes Six Sigma theory and how it relates to other change strategies. Theoretical progress, we believe, is critical to the development of Six Sigma studies. Is proposed based on a six sigma model. An analysis of variance is used to investigate the effect of model parameters and their interactions on the optimal solution. Implementing the Six Sigma approach in industries aids in the development of high-quality goods with few defects and process variations. That is, unlike the traditional control chart, the six sigma with economic statistical design chart helps to know the status of the process in terms of lev-el sigma and DPMO. This in turn helps to involve in continuous quality unknown though the sample mean is an improvement activity so that the process unbiased estimator of the population mean. Can achieve the Six Sigma quality's goal of There exist many conventional ways of 3.4 DPMO. Hence, a practice of estimating population standard deviation continuous quality improvement, it is such as sample standard deviation, sample range, etc. While this study does not pretend to be comprehensive, it does include useful information about the current state of economic statistical design analysis. One of the most important findings from our research was the increased analytical emphasis on economic statistical architecture, which has shifted to the use of Six Sigma tools in recent years. There isn't much space in the literature to explain what Six Sigma theory is and how it fits in with other improvement techniques. Theoretical progress, we believe, is critical to the development of Six Sigma studies.

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